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A CIRCULAR SANDWICH PLATE, TRANSVERSELY LOADED

A Thesis
Submitted to the Graduate Faculty
of the
University of Minnesota
by
Edward B. Johnson

In Partial Fulfillment of the Requirements
for the
Degree of Master of Science in Aeronautical Engineering

July, 1949

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NOMENCLATURE

r = radial distance

R = radius of plate

M_r, M_t = moments in radial and tangential direction

σ_r, σ_t = stresses in radial and tangential direction

ϵ_r, ϵ_t = strains in radial and tangential direction

P = uniform load

δ = deflection of plate measured from unloaded position

δ_o = deflection of the center of the plate

$(\sigma_r)_b$, $(\sigma_t)_b$ = radial and tangential bending stresses

$(\sigma_n)_r$, $(\sigma_n)_t$ = radial and tangential normal stresses

E = Young's modulus

μ = Poisson's ratio

A CIRCULAR SANDWICH PLATE, TRANSVERSELY LOADED

SUMMARY

Strain and deflection measurements were made on a simply supported, circular sandwich plate under a uniform transverse load, and the resulting stresses and flexure compared with those of a similar theoretical plate.

The plate used in this investigation was composed of two thin aluminum alloy face plates and a lightened plywood core, fastened together by means of transverse rivets extending completely through the plate.

With repect to flexure, the experimental results agree well with flat plate theory at low loading conditions, and show increasing departure from the theory as the load is increased. Radial and tangential bending stresses show good agreement with flat plate theory. For deflection analysis, this investigation indicates that flat plate theory will always yield conservative results.

In the construction of this type of plate "cherry" rivets possess several advantages over ordinary rivets. There is no problem of shank

A CIRCULAR SANDWICH PLATE, TRANSVERSELY LOADED

INTRODUCTION

The need for a structural material possessing high strength and stiffness, light weight, and smooth aerodynamic surfaces under high stress has led to the study of sandwich plate construction as a substitute for sheet-stringer construction in airplane design. A sandwich plate consists of two thin face sheets of high-strength material separated by a relatively thick low-density core. Some materials that have been used as core material are balsa wood, hard foam rubber, cellulose acetate, sheet metal corrugations, and honeycomb material made of thin sheet metal or resin-impregnated cloth or paper. The face sheete may be of metal, plywood, or some other type of high-strength material. The core and face sheets may be connected by means of rivets or by a bonding material such as a phenolic resin.

The circular plate considered in this analysis consisted of thin aluminum alloy face sheets and fir plywood core connected by aluminum rivets extending transversely through the plate.

The purpose of this investigation was to study the behavior of a sandwich plate under relatively high transverse load, and to compare the resulting

stresses and flexure with those of a similar theoretical plate.

The author wishes to express his appreciation to his adviser, Professor Joseph A. Wise, whose suggestions and assistance have been most valuable.

These tests were conducted at the University of Minnesota Engineering Experiment Station, Minneapolis, Minnesota in June, 1949.

EQUIPMENT AND PROCEDURE

The plate considered in this analysis is shown in Fig. 1. It was composed of two .025 inch 24 ST aluminum Alclad face sheets separated by a .25 inch fir plywood core which had been lightened by 1 inch holes drilled 1.25 inch from center to center as shown in Fig. 1. The face plates were connected to the core by means of 5/32 inch aluminum rivets extending transversely through the plate and evenly spaced between the lightening holes as shown in Fig. 1.

Fig. 2 shows the test equipment. The test plate was supported on a 30 inch diameter ring which had been fabricated from a one inch beaded right angle aluminum channel. The supporting ring was bolted to a 2" thick steel base. The system was

made air tight. Uniform transverse loading was accomplished by reducing the air pressure in the volume included between the base, ring, and test plate. An air aspirator was used for this purpose. Loading pressure was measured by a mercury manometer.

Strains were measured by means of SR-4 electrical resistance strain gauges, and deflections by means of Ames dials. The arrangement of these instruments is shown in Fig. 3.

All strain measurements were made at a load of 4 #/in^2. Deflections were measured at pressures varying from 2.4 to 5 pounds per square inch.

RESULTS AND DISCUSSION

The theory for the bending of an isotropic plate, the section transformation of the sandwich material, the development of a boundary parameter, and application to the subject plate are given in Appendix A. The basic theory assumes ideally elastic, homogeneous, isotropic material and neglects any deformation due to shear, and the effect of any deformation of the middle surface.

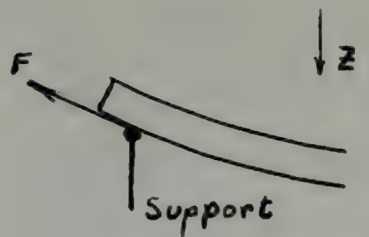
Fig. 4 shows that at a load of 4 pounds per square inch the deflected surface of the experimental plate has the same form as the theoretical

plate, but that the amount of deflection is less. The application of Olson's[3] correction shows close agreement between the experimental and theoretical plates. Fig. 5, a plot of center deflection as a function of load, shows that at the lower loading conditions the experimental deflection agrees very well with plate theory and that as the load increases the departure from theory also increases. This fact is also illustrated in Figs. 6,7, and 8.

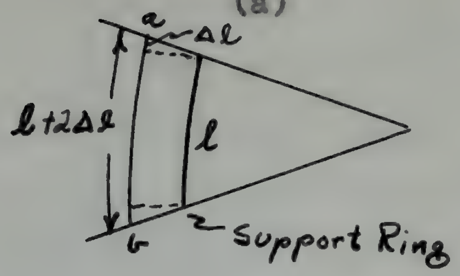
Strain measurements for a load of 4 pounds per square inch are given in Table II; the resulting two dimensional state of stress in Table III. Figs. 9 and 10 show the variation of radial and tangential stresses of the upper and lower plate surfaces as a function of radial distance. These plots show that the experimental stresses of the lower surface are higher than the theoretical stresses; the upper surface stresses, lower. A resolution of the measured radial and tangential stresses into radial and tangential bending and normal stresses is shown in Figs. 11, 12, 13, and 14. Here the experimental bending stresses show good agreement with the theory.

The support used in these tests allows rotation but prevents displacement in the direction. There exists at the support a frictional force F, sketch

(a), which increases as the load increases. This force causes a radial tension in the plate.



(a)



(b)

This force, which is $(\sigma_n)_r$, Fig. 13 also causes the plate deflection to be less than it would be if the axial force were not present. Since no axial force is assumed in the theory, this accounts for the difference between the experimental and the theoretical deflections.

The arc ab (sketch b) which is located just outside of the support ring in the unloaded condition will move over the ring when the plate is loaded. In so doing it will decrease in length an amount 2 , and cause a tangential compressive strain. This strain decreases as the distance from the plate center decreases, and is zero at the center. $(\sigma_n)_t$ must equal $(\sigma_n)_r$ at the center, and will decrease as r increases, and finally take on compressive values in the region of the support. This behavior is shown in Fig. 15.

CONCLUSIONS AND RECOMMENDATIONS

Experimental bending stresses show good agreement with plate theory. Deflection measurements show good agreement with theoretical plate deflections at low loading conditions. As the load is increased the departure from theory becomes more pronounced; the experimental deflection curve has less curvature than the theoretical curve. This is due to support friction which produces a radial force in the plate, and causes the plate to appear to have greater stiffness than it actually possesses.

It is believed that the subject type of sandwich construction possesses sufficient potentiality as an engineering material to warrant further investigation and comparison with other types of plates.

In view of fabrication experience, the use of "Cherry" type rivets is recommended. These rivets possess several advantages over ordinary rivets, including: (1) the elimination of shank buckling and the resultant possibility of using other core material, such as balsa wood; (2) the sandwich plate may be countersunk on both sides, thus presenting flush surfaces; (3) the "Cherry" rivet draws the material as it is set, thus removing the probability of face plate damage during the drawing-driving operation of ordinary rivets.

TABLE I

PLATE DEFLECTIONS

For P = 4 /in^2

Sta	1	2	3	4	5	6	7
r_1	12 $\frac{15}{16}$	7$\frac{1}{4}$	3 $\frac{7}{8}$	0	3	6$\frac{1}{4}$	11 $\frac{1}{8}$
r_2	12 $\frac{3}{8}$	7	3 $\frac{13}{16}$	0	2 $\frac{15}{16}$	6 $\frac{1}{8}$	11 $\frac{3}{8}$
δ_1	.151	.591	.725	.798	.765	.658	.295
δ_2	.230	.630	.736	.800	.770	.653	.302

Deflection at Center (δ_o) for Various P

Run #1

P	4.0	3.37	3.08	2.64	2.52	2.34
δ_o	.80	.740	.709	.66	.638	.606

Run #2

P	4.83	4.38	4.0	3.66	3.22	2.88	2.46	2.48
δ_o	.908	.85	.80	.757	.730	.702	.678	.650

Deflectiom at Different Stations for Various P

P,#/in^2	r	12	7$\frac{1}{4}$	3	0	3	6$\frac{1}{4}$	11
3.37	δ	.136	.541	.673	.737	.690	.608	.264
3.08	"	.126	.541	.635	.700	.654	.575	.244
2.63	"	.110	.474	.595	.641	.606	.534	.219
2.51	"	.102	.457	.573	.625	.578	.512	.207
2.34	"	.095	.441	.554	.605	.565	.496	.198

TABLE II

STRAIN MEASUREMENTS

Run		I	II	III
Lead	Dist			
1	1"	-600	-591	-604
2	3/4	+1390	+1395	+1380
3	1	+150	+160	+142
4	3/4	+765	+770	+779
5	1	-280	-265	-285
6	3/4	+1145	+1153	+1155
7	3 3/8	-615	-613	-624
8	3 3/8	+1335	+1319	+1334
9	3 3/8	-815	-923	-927
10	3 3/8			
11	7 1/8	-300	-289	-292
12	7 1/8			
13	7 1/8	+55	+61	+60
14	7 1/8	+725	+730	+733
15	7 1/8	-355	-343	-346
16	7 1/8	+885	+882	+877
17	10 5/8	-80	-90	-91
18	10 5/8			+996
19	10 5/8	-270	-269	-285
20	10 5/8	+270	+272	+273
21	14 7/8	+495	+500	+506
22	14 3/8	+640	+646	+678
23	14 7/8	-770	-748	-750
24	14 3/8	-470	-468	-435
25	3¼	-800	-810	-809
26	3¼	+1375		+1230
27	3¼	-645	-659	-660
28	3¼	+1520	+1626	+1622
29	7 1/8	-40	-38	-42
30	6 7/8	+1150	+1156	+1160
31	7 1/8	+190	+210	+190
32	6 7/8	+420	+432	+440
33	7¼	-310	-302	-311

TABLE II (Cont'd.)

34	6 7/8	+465	+462	+480
35	10 3/4	-110	-114	-115
36	10 3/4	+1220	+1214	+1220
37	10 3/4	-1050	-1047	-1042
38	10 3/4	+645	+645	+637
39	14 3/4		+515	+519
40	14 3/8	+612	+662	+787
41	14 3/4	-900	-870	-871
42	14 3/8	-290	-286	-279
43	6 7/8	-330	-330	-330
44	6 7/8	-932	-919	-916
45	14 3/4	+775	+770	+770
46	14 3/4		-976	-960

TABLE III

Solution of $\sigma_r = \frac{E}{1-\mu^2}(\epsilon_1 + \mu\epsilon_2)$ where $E = 10^7$, $\mu = .25$

Sta	(σr) upper $\epsilon_1\ \epsilon_2$	$(\sigma r)_u$	(σr) lower $\epsilon_1\ \epsilon_2$	$(\sigma r)_l$	(σ_t) upper $\epsilon_1\ \epsilon_2$	$(\sigma t)_u$	(σt) lower $\epsilon_1\ \epsilon_2$	$(\sigma t)_l$	radial $(\sigma r)_b$	$(\sigma r)_n$	tangential $(\sigma t)_b$	$(\sigma t)_n$
(r)												
A 1"	1, 5	+7200	2,6	+17800	5,1	∓4650	6,2	+16000	±11225	+6575	±10325	+5675
B $3\frac{1}{4}$	7, 9	+9150	8,10	-	9,7	∓11500	10,8	-	-	-	-	-
C $7\frac{1}{8}$	11,15	+4040	12,16	-	15,11	∓4450	16,12	-	-	-	-	-
D $10\frac{5}{8}$	17,19	+1230	18,20	+11050	19,17	∓3280	20,18	+5470	±6140	+4910	±4375	+1095
E $14\frac{7}{8}$	21,23	+3400	22,24	+6090	23,21	∓6620	24,22	∓2820	±1345	+4745	±1900	+4720
F 1"	5, 1	+4650	6,2	+16000	1,5	∓7200	2,6	∓17800	±10325	+5675	±12000	+4800

TABLE III (Cont'd.)

F	3¼	25,27	-10800	26,28	+17400	27,25	-8790	28,26	+20500	±14100	+3300	±14645	+5855
G	7 1/8	29,33	-1270	30,34	+13650	33,29	-3300	34,30	+8200	±7560	+6290	±5750	+2450
H	10 5/8	35,37	-4040	36,38	+14700	37,35	-11800	38,36	+10000	±9370	+5330	±10900	-900
I	14 7/8	39,41	+3200	40,42	+7650	41,39	-7890	42,40	-1210	±2225	+5425	±3340	-4550

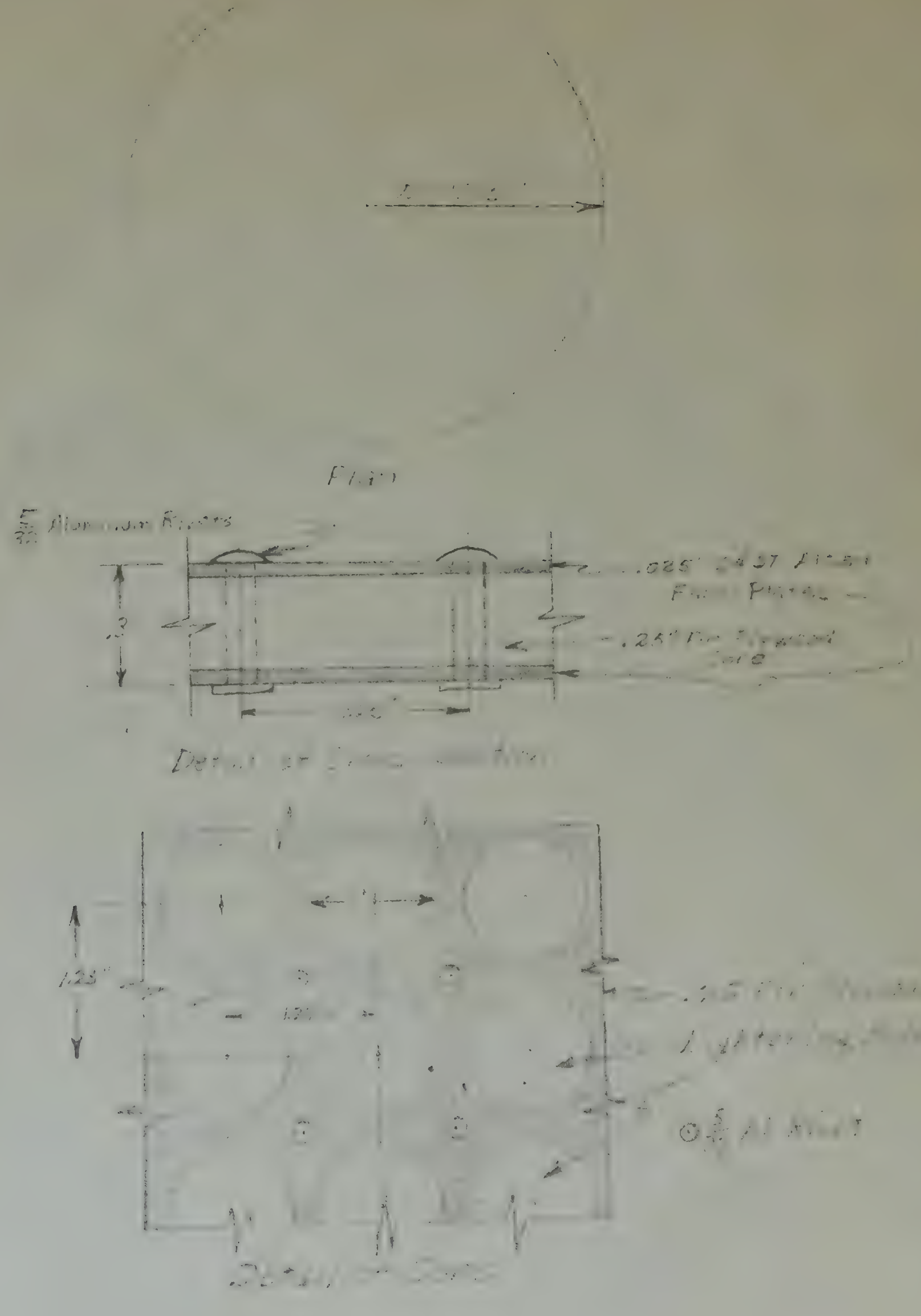


Fig. 1. End Plate

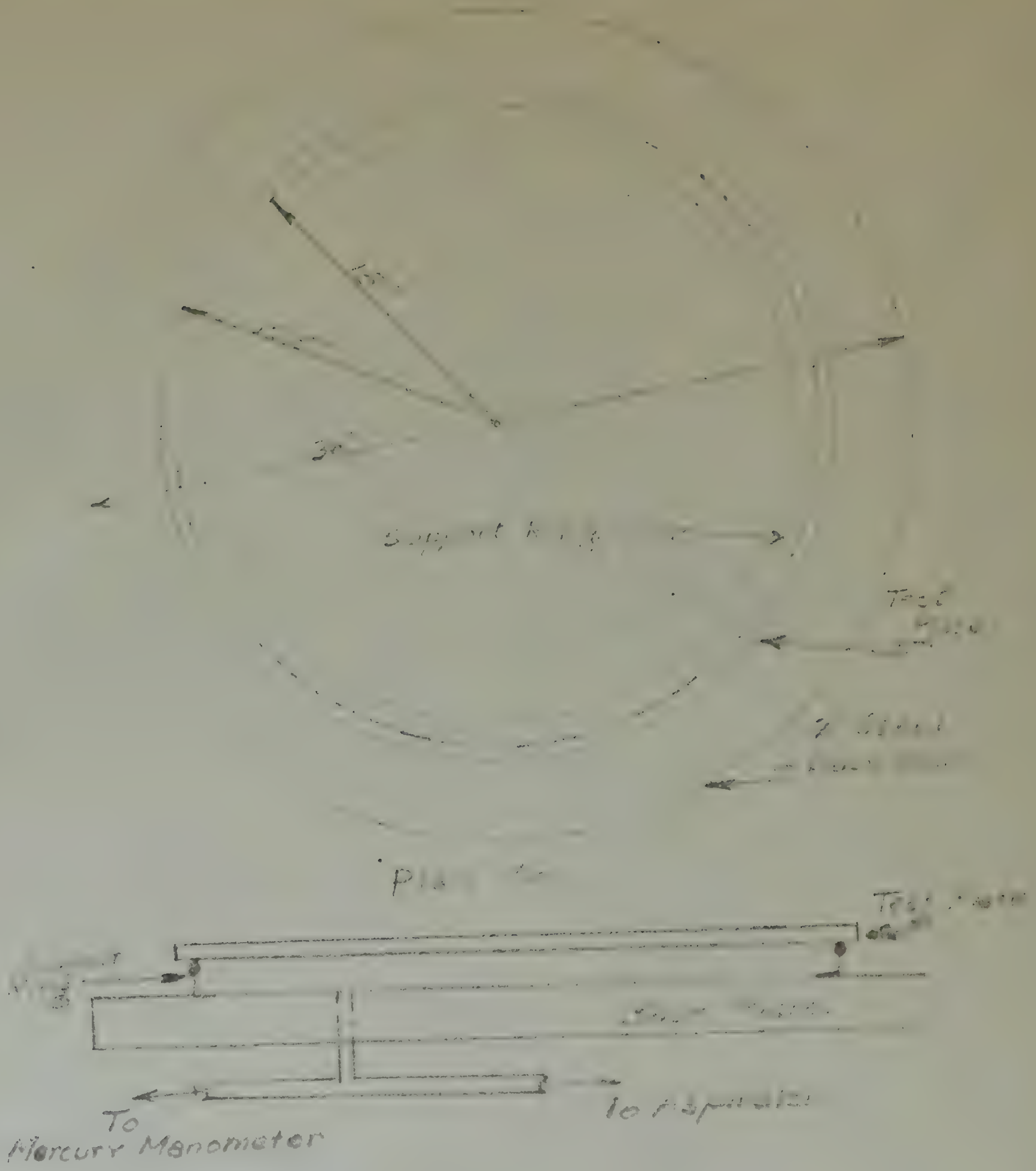
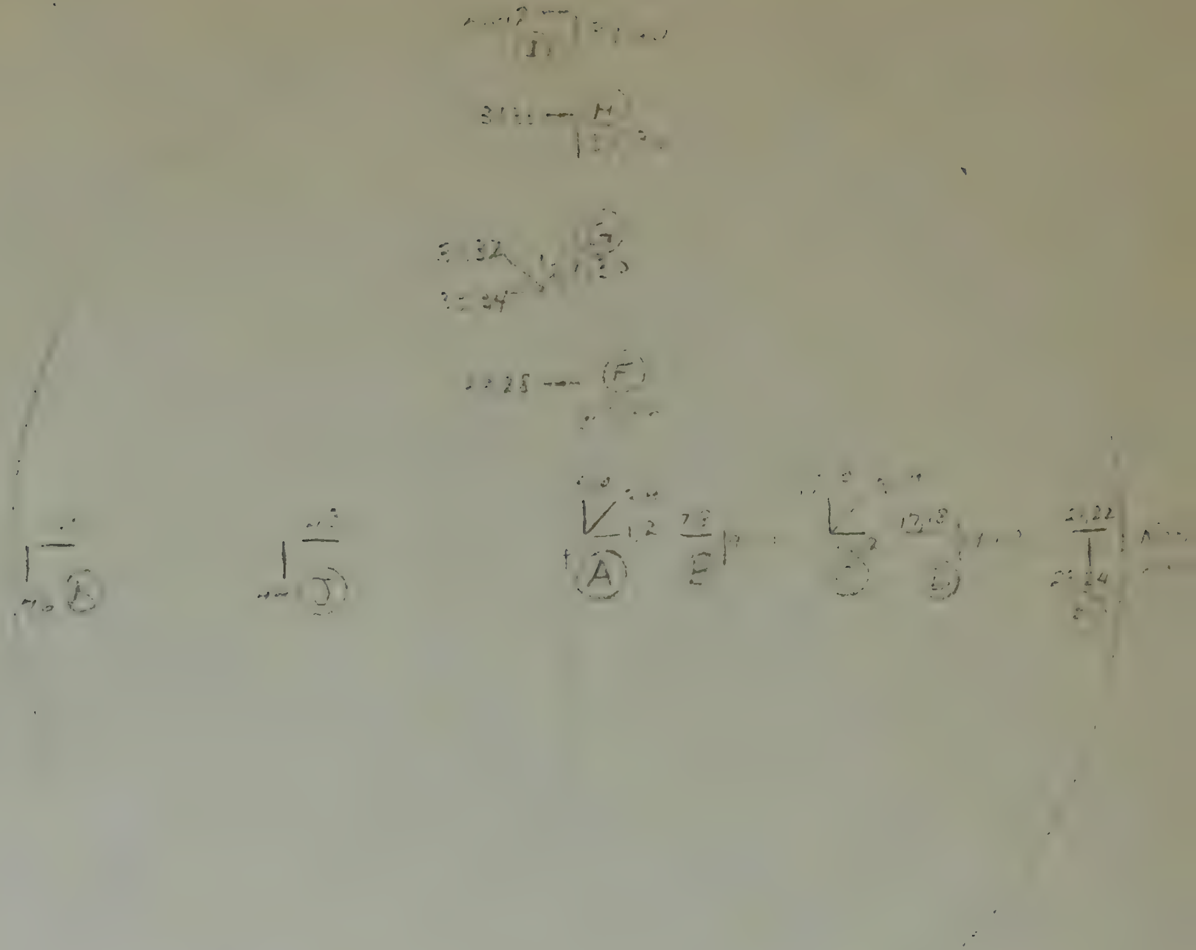


Fig. 2. Test Set-up



[illegible]: (1) Numbers denote strain gauge lead; letters denote station. Odd numbers are on top (compression) side; even numbers are on the bottom (tension) side, except Nos. 44 and 46 which are on top.
(2) Letters denote strain gauge station.
(3) Types of strain gauges:
AR-1 - Sta. A, C, and E
A-8 - Sta. F, I, and K
A-11 - All other stations
(4) Ames dials were mounted above Ref Axis.
(5) All radial distances from center, r, are given in Tables I and II.

Fig. [illegible]. Strain Gauge and Ames Dial Arrangement

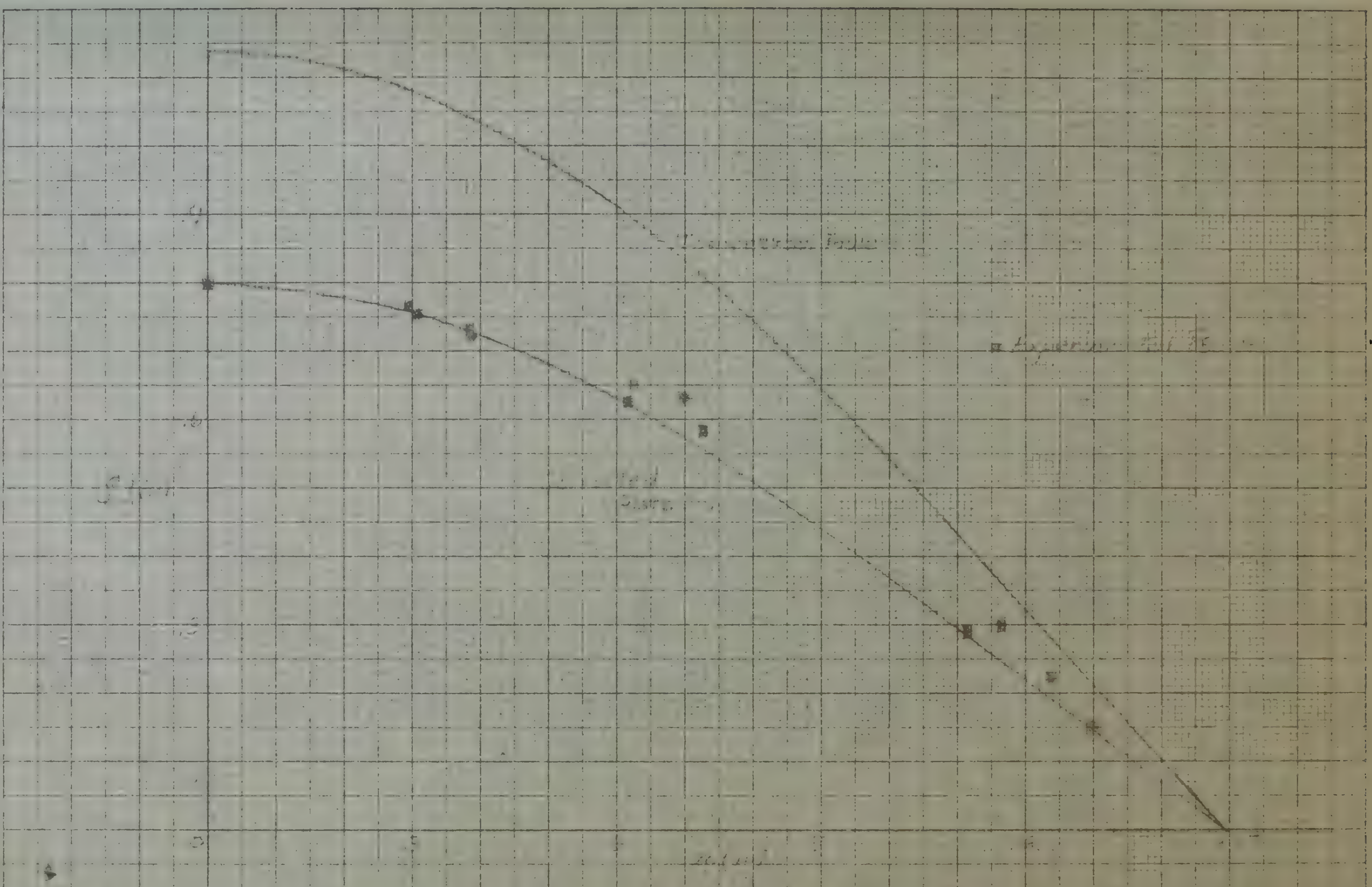


Fig. [illegible]. Lateral Deflection (S) vs. Initial Distance to [illegible] for
Test (P) of [illegible]

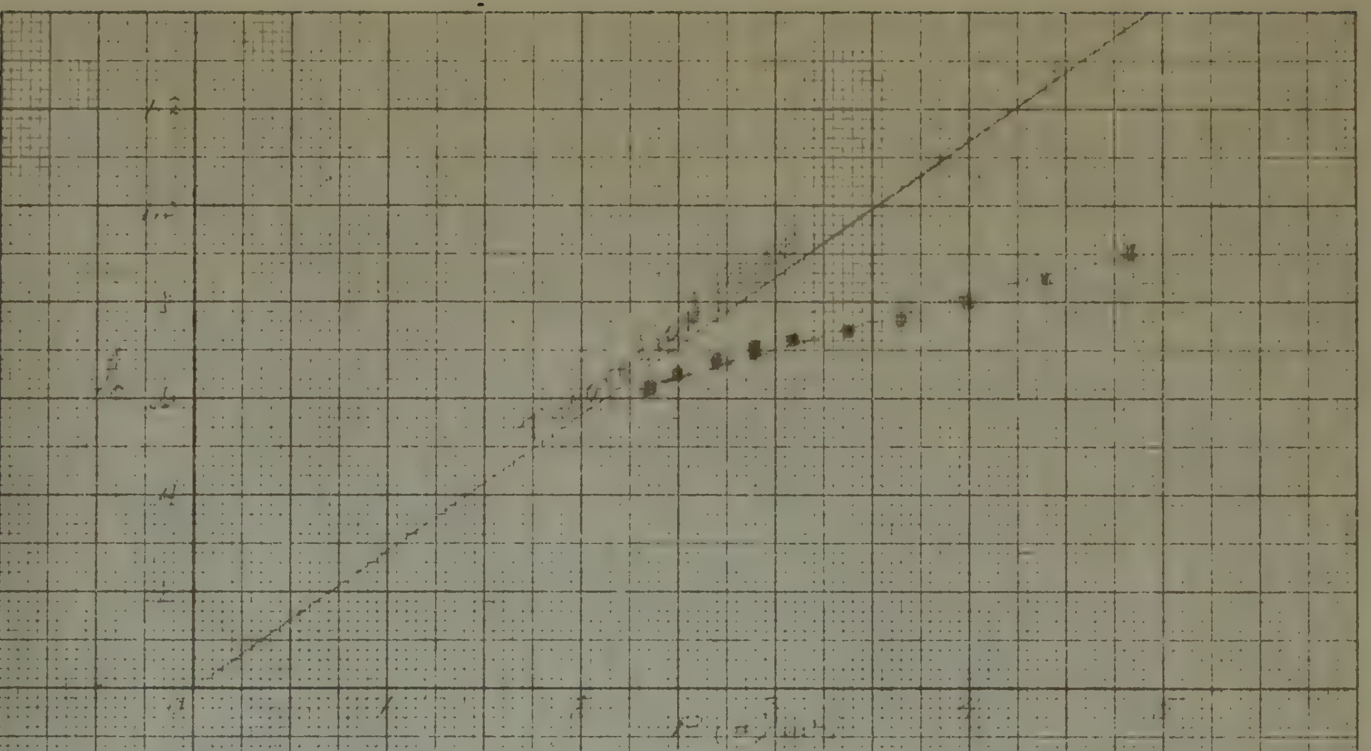


Fig. 5 Center Defl. [illegible] Same Load

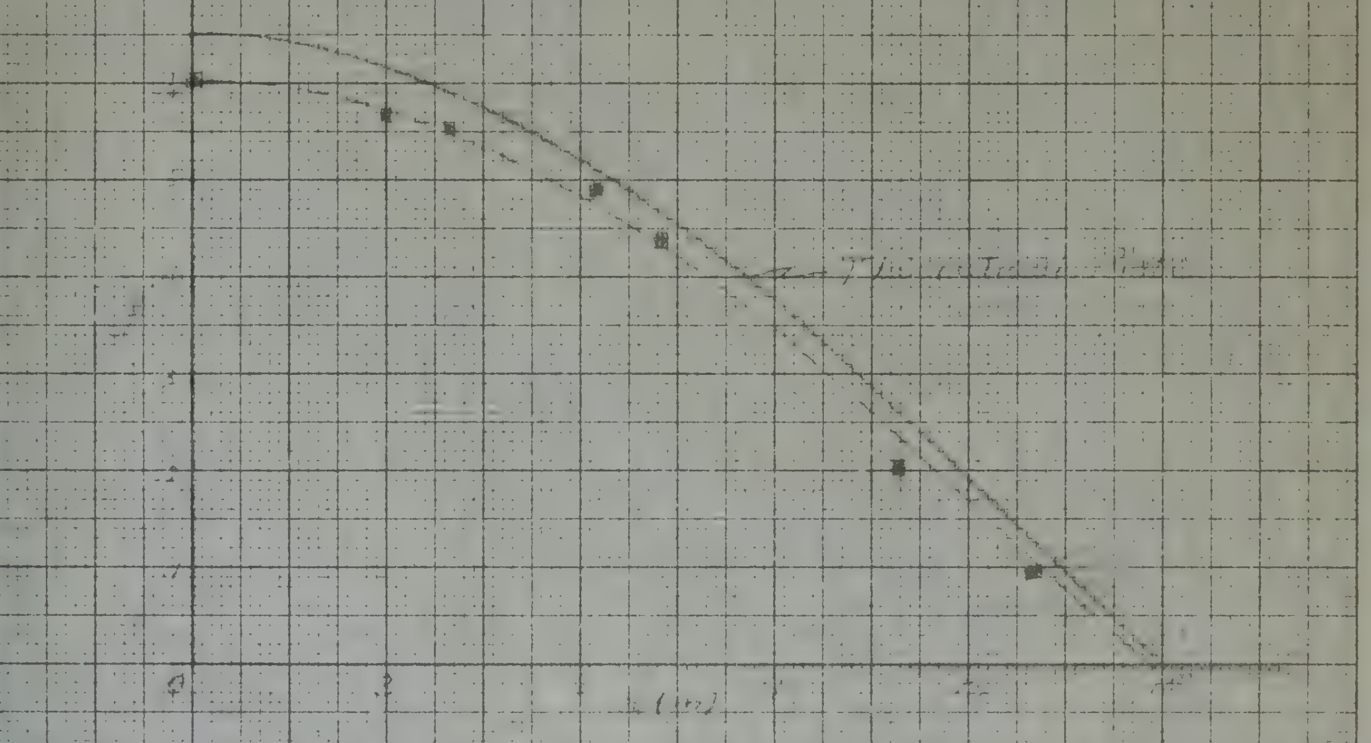


Fig. 6 Deflection at Radial [illegible] for P = 2.34 [illegible]

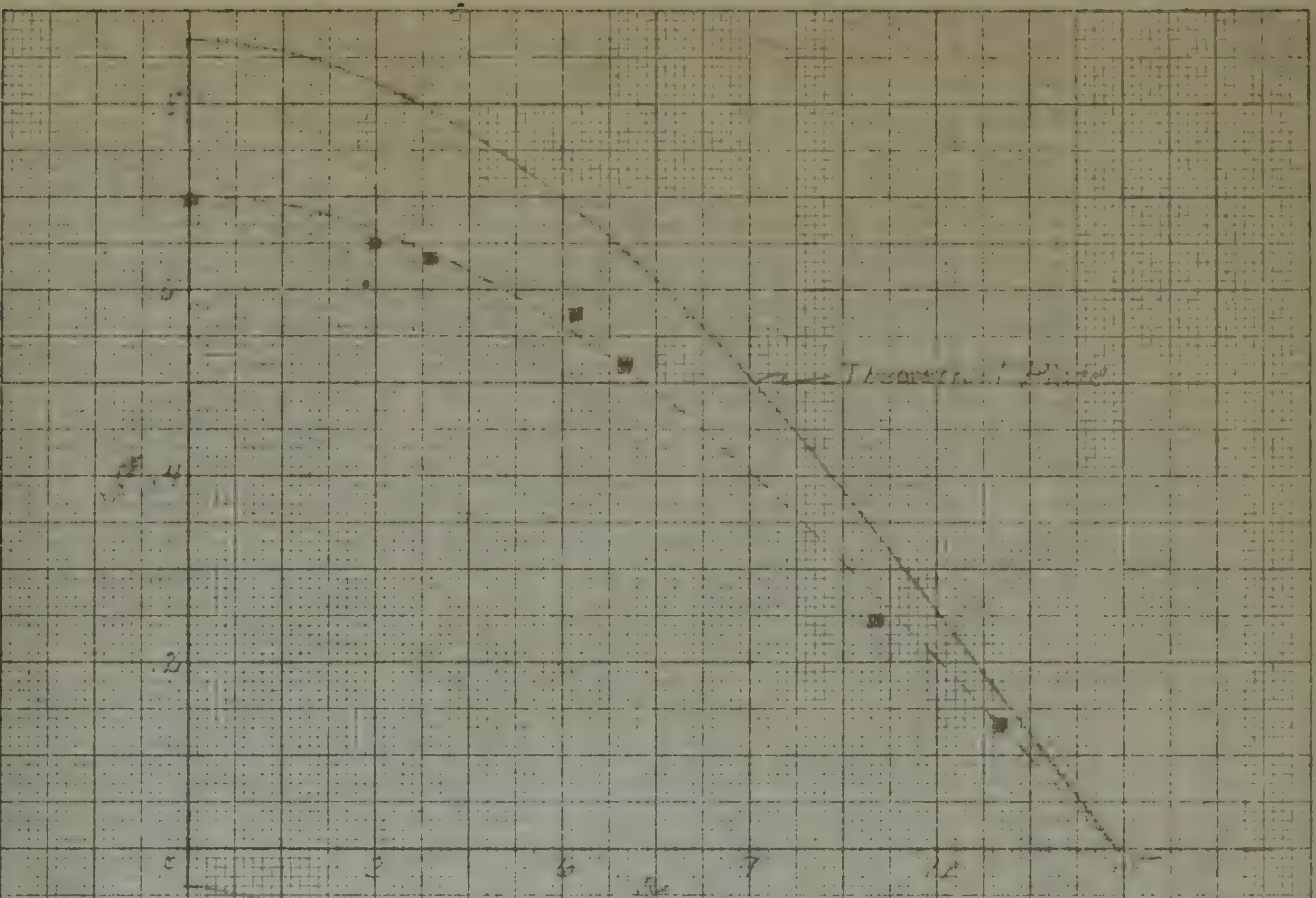


Fig 7 [illegible] F. 2 [illegible]

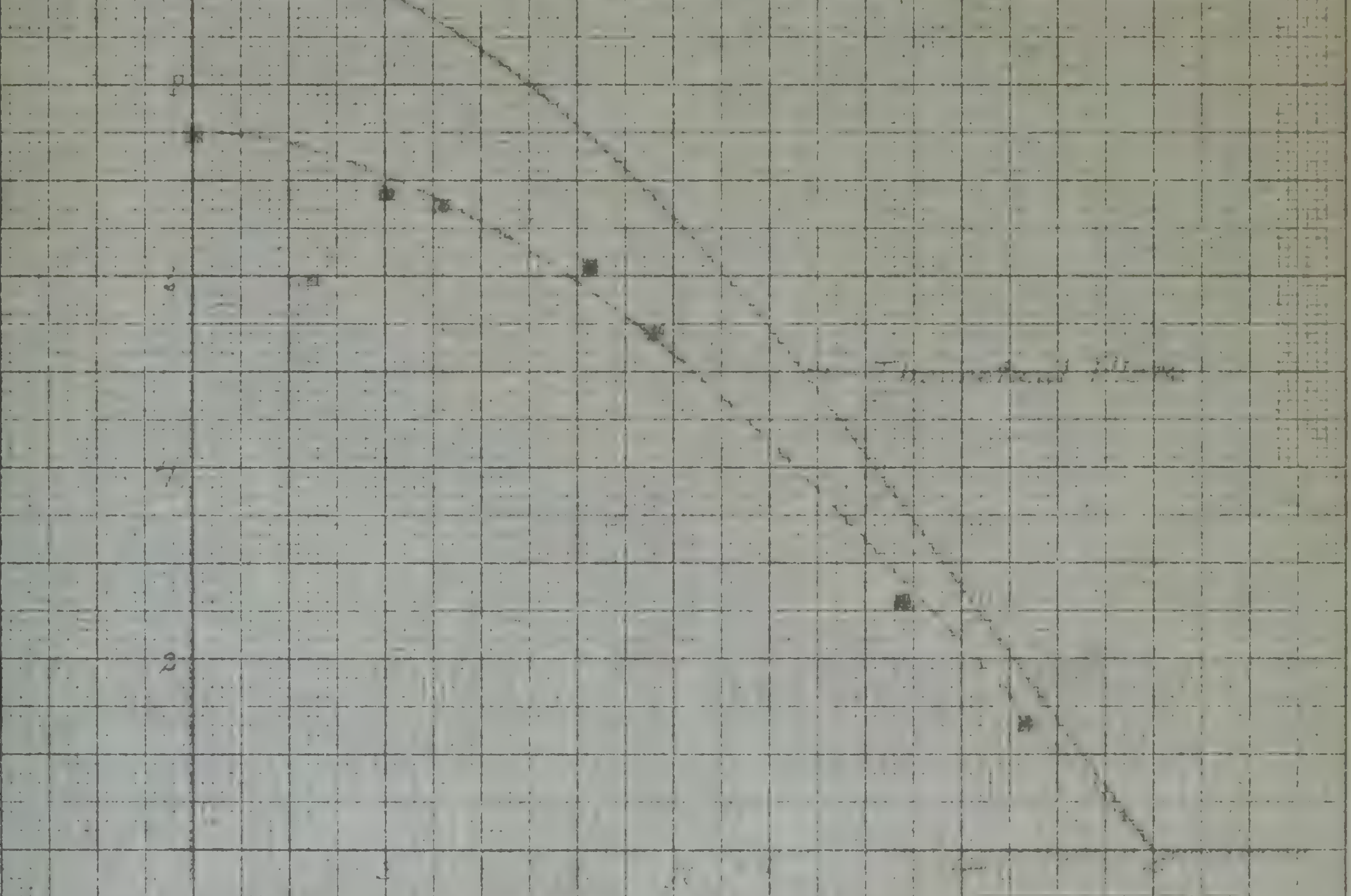


Fig 8 [illegible] F. 27

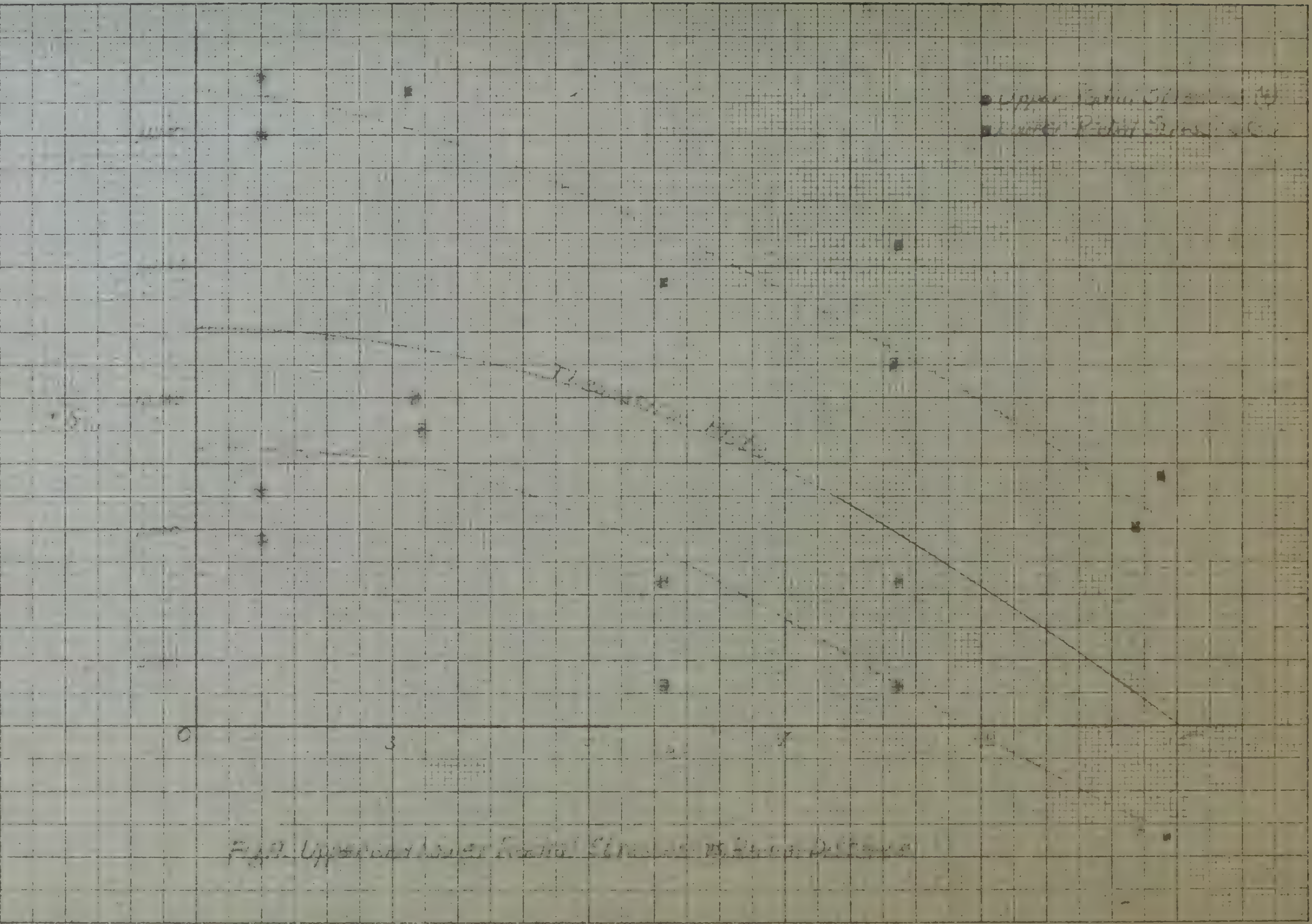


Fig. 18. Upper and Lower Radial Stresses vs Beam Distance

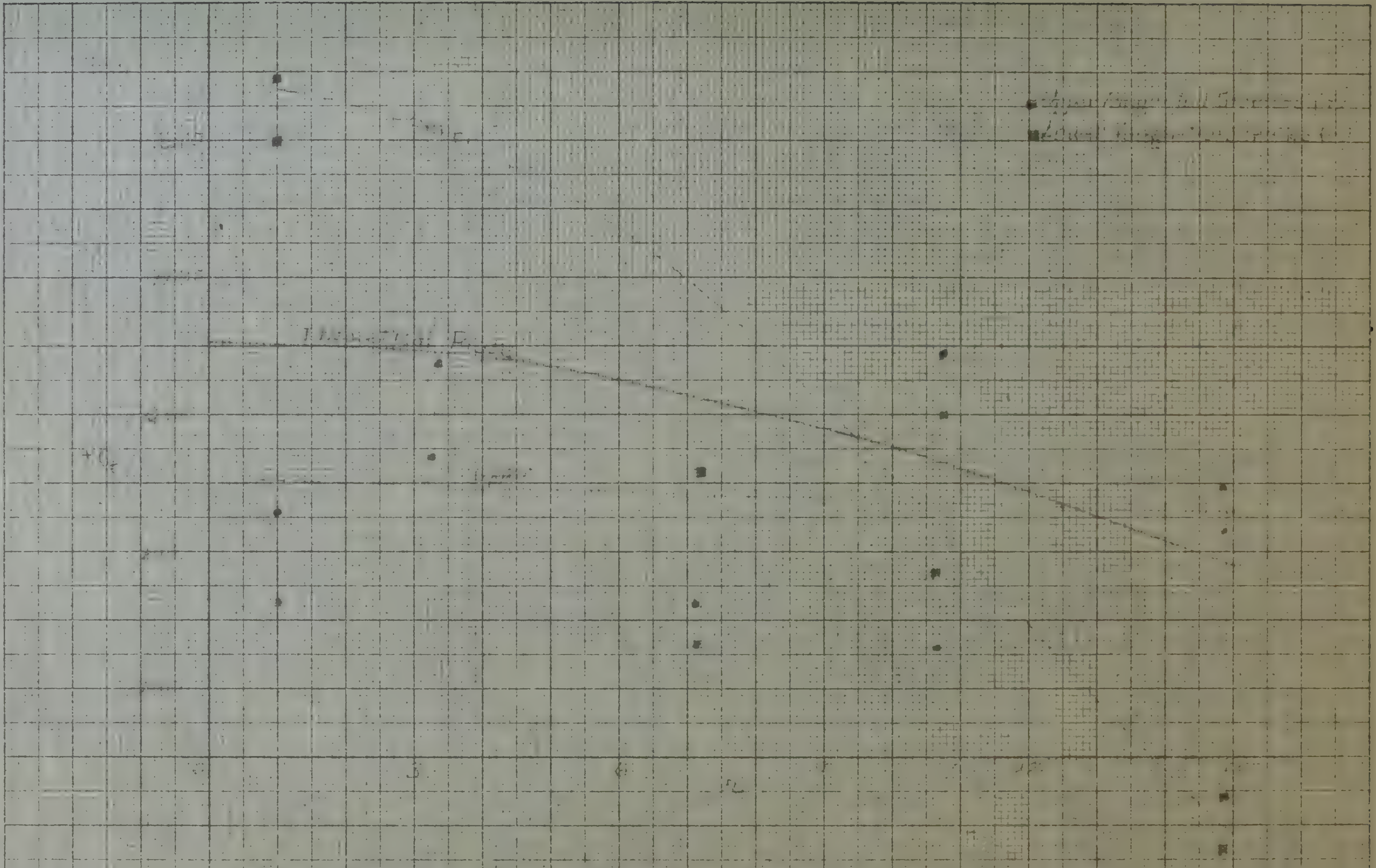


Fig. 16 Upper and Lower Tangential Stresses vs Neutral Distance

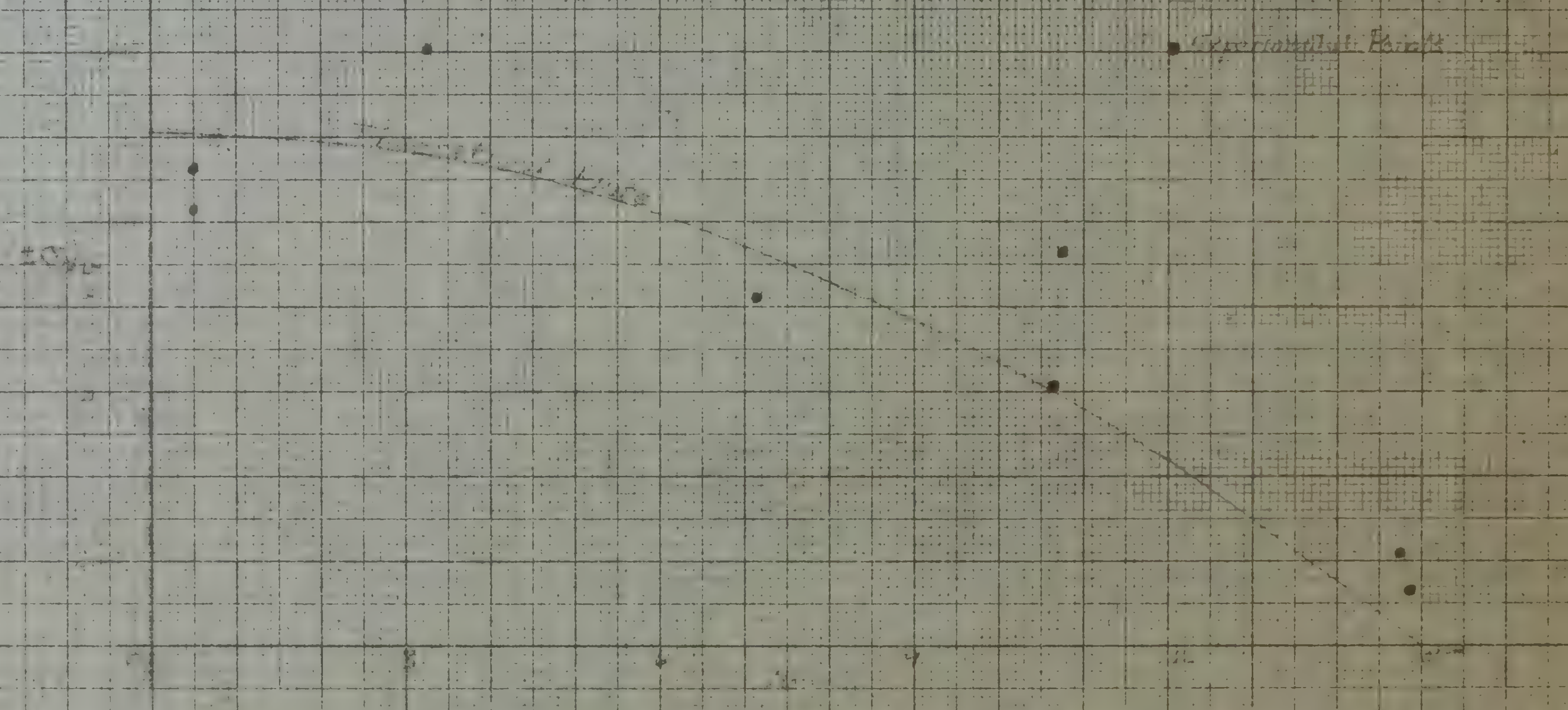


Fig. 4 Radial Bending Stresses vs. Radial Distance

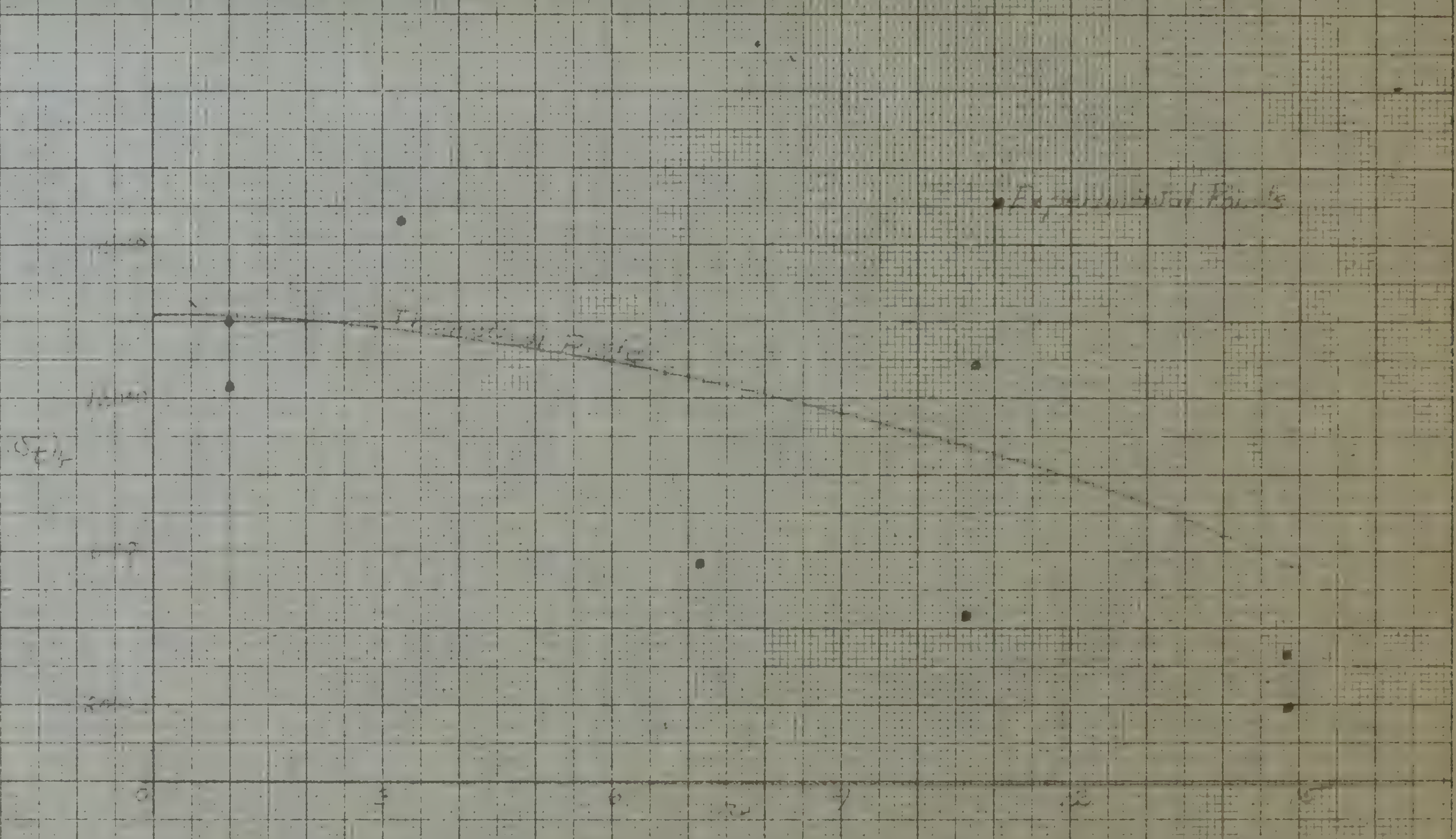


Fig. 12 Theoretical Bending Stress vs. [illegible]

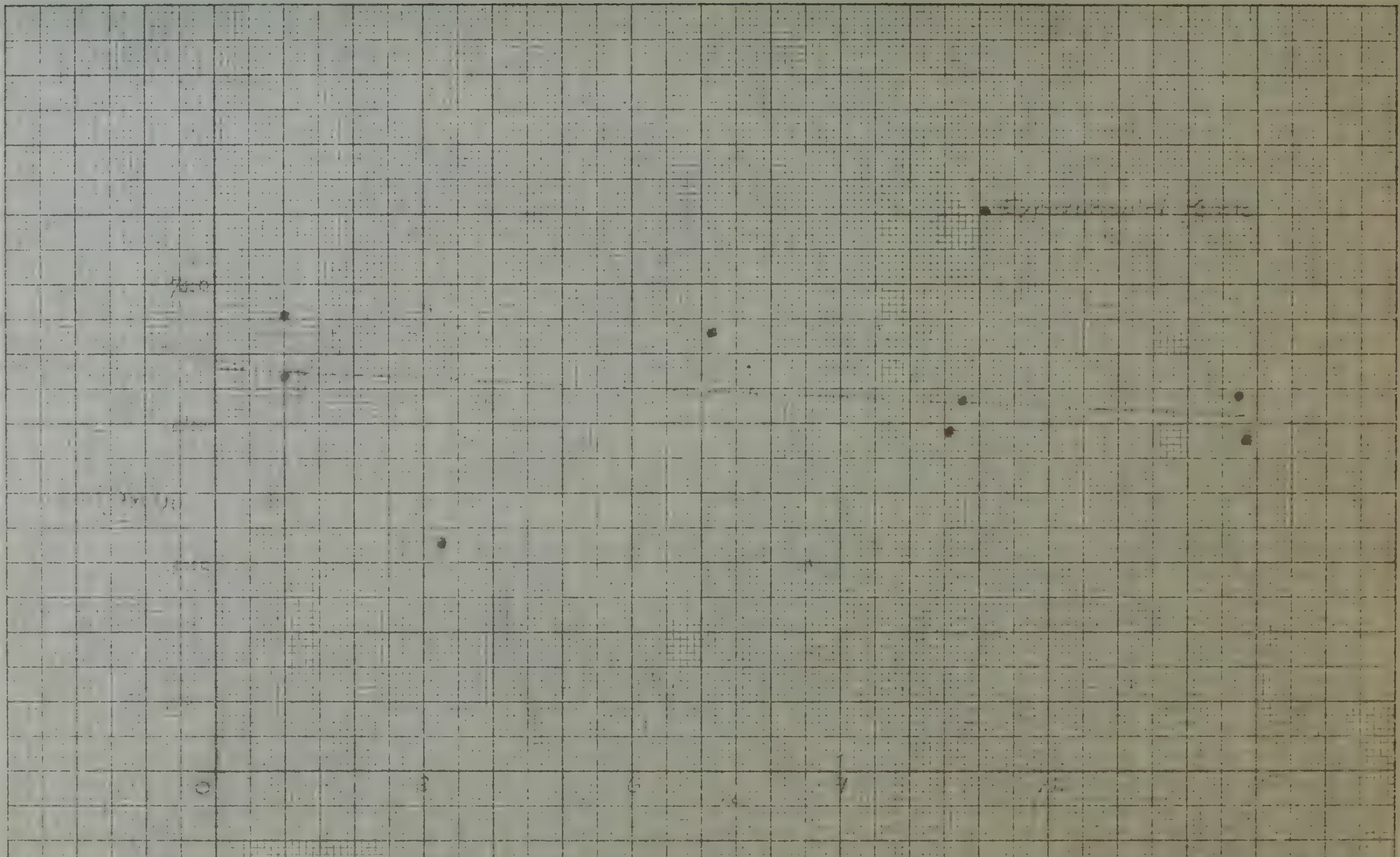


Fig 13 Normalized Stress vs Radial Distance

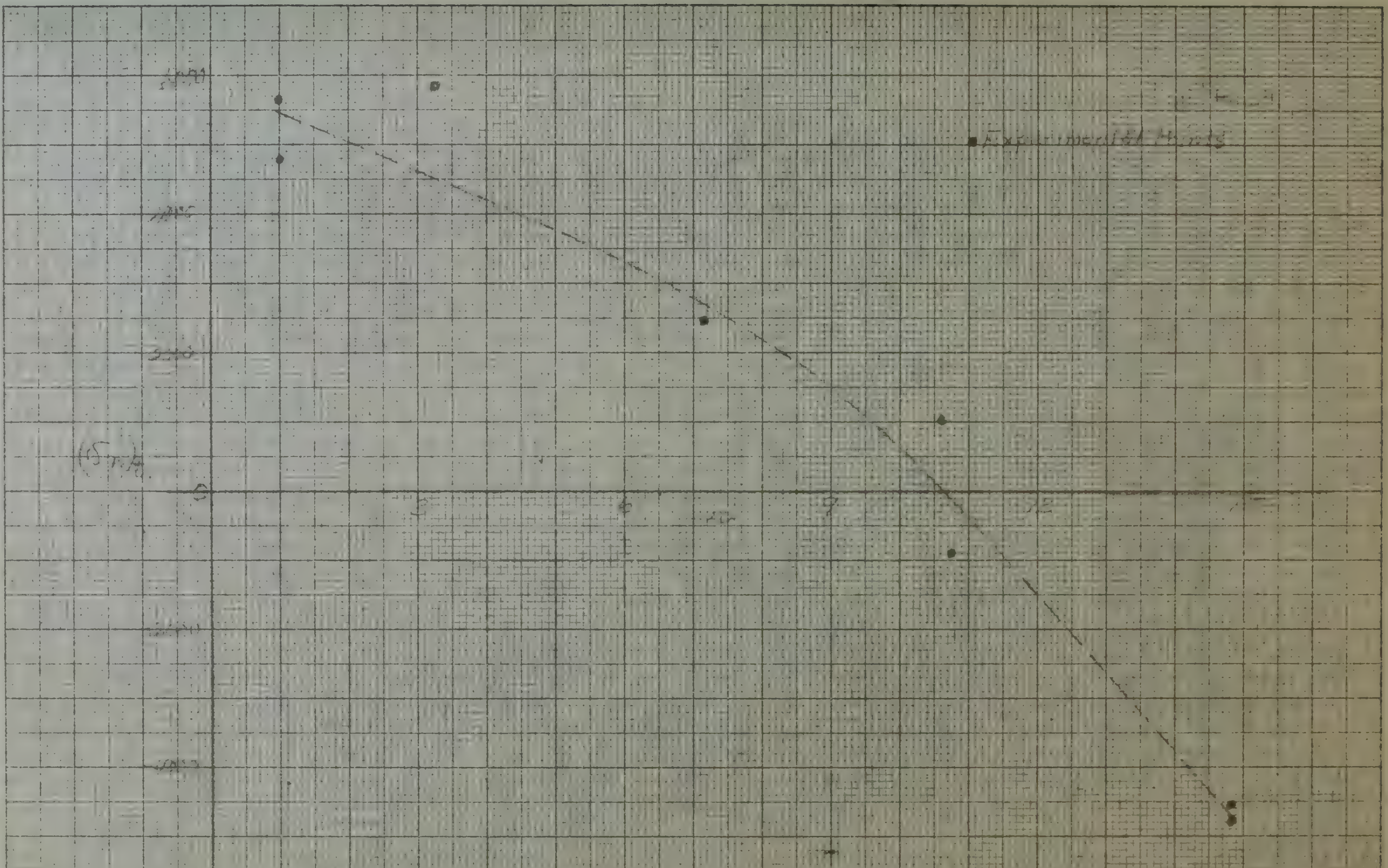
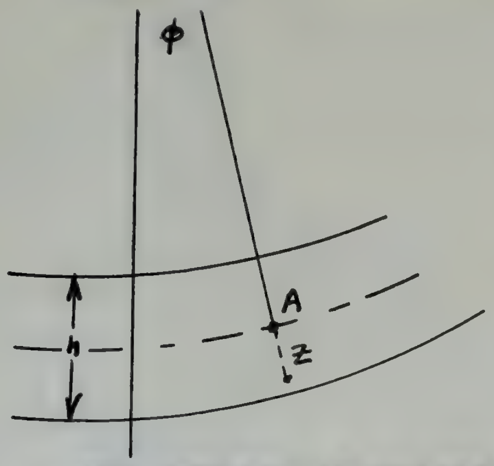


Fig 4 Normal Tangential Stress vs Radial Distance

APPENDIX A

Bending of a Circular Plate Loaded Symmetrically with Respect to the Center. This development follows that of Timoshenko[1]. It is assumed that the plate is an ideally elastic, homogeneous and isotropic solid; that straight lines normal to the neutral surface before bending remain straight and normal to that surface after flexure; that points on the neutral surface move only in a direction normal to the original plane of the neutral surface; that deflections are small compared to the dimensions of the plate; and that the stresses are a linear function of the distance from the neutral surface.



At point A, a distance r from the reference pole, $\phi = -\frac{d\delta}{dr}$. The radial curvature is

$$\frac{1}{r_1} = \frac{d\phi}{dr} = -z\frac{d^2\delta}{dr^2}$$

The tangential curvature is

$$\frac{1}{r_2} = \frac{\phi}{r} = -\frac{z}{r}\frac{d\delta}{dr}$$

The strains at r point a distance from the neutral surface are:

$$\epsilon_r = \frac{z}{r_1} = -z\frac{d^2\delta}{dr^2}$$

$$\epsilon_t = \frac{z}{r_2} = -\frac{z}{r}\frac{d\delta}{dr}$$

The stresses are:

$$\sigma_r = \frac{E}{1-\mu^2}(\epsilon_r+\mu\epsilon_t) = \frac{Ez}{1-\mu^2}\left(\frac{d\phi}{dr}+\mu\frac{\phi}{r}\right) \qquad (1)$$

$$\sigma_t = \frac{E}{1-\mu^2}(\epsilon_t+\mu\epsilon_r) = \frac{Ez}{1-\mu^2}\left(\frac{\phi}{r}+\mu\frac{d\phi}{dr}\right) \qquad (2)$$

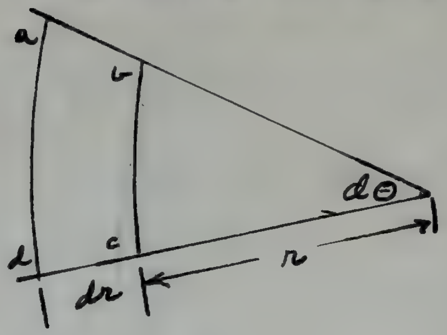
The moments per unit of width are:

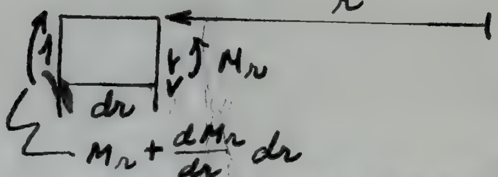
$$M_r = D\left(\frac{d\phi}{dr}+\mu\frac{\phi}{r}\right) \qquad (3)$$

$$M_t = D\left(\frac{\phi}{r}+\mu\frac{d\phi}{dr}\right) \qquad (4)$$

Where D $= \frac{Eh^3}{12(1-\mu^2)} = \frac{EI}{1-\mu^2}$

Considering the equilibrium of an element abcd,





The couple acting on cd is,

$$M_r r d\theta$$

The couple acting on ab is,

$(M_r + \frac{dM_r}{dr}dr)(r+dr)d\theta.$

The couples on sides ad and bc are $M_t dr$, which in the radial direction are, $M_t dr\, d\theta$. If V = shearing force per unit of length, the shearing couple is, $V_r\, d\theta\, dr$. Summing these moments,

$$(M_r + \frac{dM_r}{dr}dr)(r+dr)d\theta - M_r r d\theta - M_t dr\, d\theta + V_r\, d\theta\, dr = 0.$$

Neglecting small higher order terms,

$$M_r + \frac{dM_r}{dr}r - M_t + V_r = 0. \qquad (5)$$

Substituting (3) and (4) in (5).

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} - \frac{\phi}{r^2} = -\frac{V}{D} \qquad (6)$$

For a uniform load, P, the load acting at a radius r, is $\pi r^2 P$, the shearing force per unit of length, V, is

$$V = \frac{\pi r^2 P}{2 \pi r} = \frac{Pr}{2}$$

then

$$\frac{d^2 \phi}{dr^2} + \frac{1}{r} \quad \frac{d\phi}{dr} - \frac{\phi}{r^2} = \frac{-1}{D}\left(\frac{Pr}{2}\right)$$

or

$$\frac{d}{dr}\left(\frac{1}{r} \times \frac{d}{dr}(r\phi)\right) = -\frac{1}{D}\left(\frac{Pr}{2}\right)$$

integrating,

$$\frac{1}{\phi r} \times \frac{d}{r}(r\phi) = -\frac{1}{D}\left(\frac{Pr^2}{4}\right) + C_1$$

$$r\phi = -\frac{1}{D} \quad \frac{Pr^4}{16} + \frac{C_1 r^2}{2} + C_2$$

$$\phi = -\frac{Pr^3}{16D} + \frac{C_1 r^2}{2} + C_2/r = \frac{-d\rho}{dr}$$

or

$$\rho = \frac{Pr^4}{64D} - \frac{C_1 r^2}{4} - C_3 + C_2 \ln r \qquad (7)$$

For a simple support,

at $r = 0$ ρ is finite, therefore $C_2 = 0$

at $r = R$, $\rho = 0$ and $M_r = 0$.

this gives

$$\rho = \frac{P}{64D}\left[\frac{5+\mu}{1+\mu} R^4 - 2\,\frac{3+\mu}{1+\mu} R^2 r^2 + r^4\right] \quad (8)$$

and

$$\sigma_r = \frac{3P}{8h^3} (3+\mu) \left[a^2 - r^2 \right] \qquad (9)$$

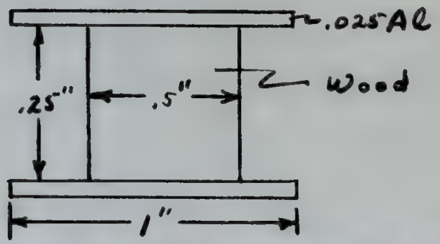
$$\sigma_t = \frac{3P}{8h^3} \left[(3+\mu)a^2 - (1+3\mu) r^2 \right] \qquad (10)$$

Equations (8), (9) and (10) are the equations used to compute the theoretical deflections and stresses in this problem.

SECTION TRANSFORMATION. The method of section transformation is that given by Niles and Newell[2].

For the core (Fig. 1) the area for a 1.25 in this square is, $1.25^2 - 3.14 \times .25 = .78$ sq. in. average width is, $\frac{.78}{1.25} = .624 \times \frac{1}{1.25} = .50$ in.

For the section



$$I_{wood} = \frac{.5 \times .25^3}{12} = .000652$$

$$I_{Al} = 2\left\{ \frac{.025^3}{12} + .1375 \times .025 \right\}$$

$$= .0009476$$

$$\frac{E_{Al}}{E_{wood}} = \frac{10^7}{10^6 \times 1.2} = 8.33$$

$$I_T = .0009476 + \frac{I_w}{8.33} = .001026$$

DEFLECTION AND STRESS CORRECTIONS BY MEANS OF SUPPORT PARAMETER[3]

This parameter is determined by the center deflection and properties and dimensions of the plate.

Let $x = \frac{r}{R}$

The equation of the surface becomes,

$$\wp = \frac{PR^4X^4}{64D} + C_1R^2X^2 + C_3 \tag{1}$$

Let $\wp = 0$ at $x = 1$ and define

$$\alpha = \frac{PR^4}{64D\wp_0} \tag{2}$$

Then (1) becomes

$$\wp = \wp_0\left(\alpha x^4 - (1+\alpha)x^2 + 1\right) = \wp_0\left(1 - x^2\right)\left(1 - \alpha x^2\right) \tag{3}$$

The bending moments then become,

$$M_r = -\frac{2\wp D}{R^2}\left[2\alpha(3+\mu)x^2 - (1+\mu)(1+\alpha)\right]$$

$$M_t = -\frac{2\wp D}{R^2}\left[2\alpha(1+3\mu)x^2 - (1+\mu)(1+\alpha)\right]$$

It can be shown that if, $\alpha = 1$, the edge is clamped, and, if $\alpha = \frac{1+\mu}{5+\mu}$, the edge is simply supported.

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